

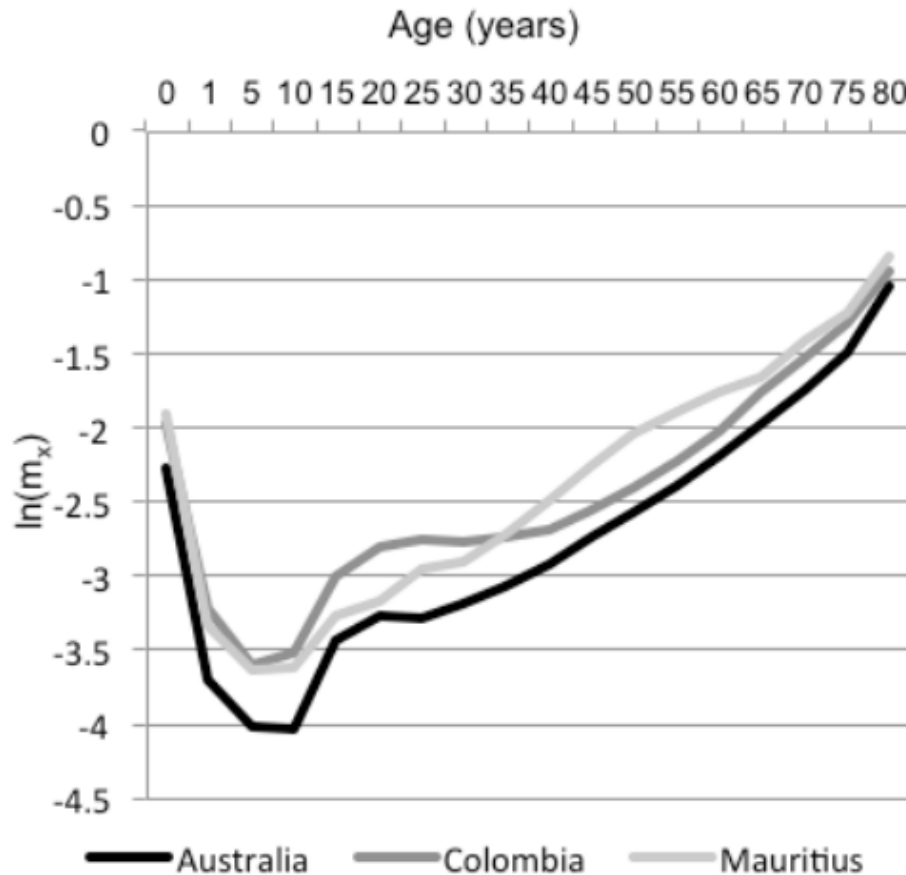


**Get
every one
in the picture**

Summary measures of mortality – **Life expectancy and life tables**

Data analysis and Report writing
workshop for Civil registration and
vital statistics data.

Recall age-specific death rates



Source: Institute for Health Metrics and Evaluation database

Graph from: University of Queensland Health
Information Systems
Knowledge Hub's *Mortality statistics: a tool to improve understanding and quality*

| Age group | Male deaths | Male population | Male Age-specific mortality rates |
|--------------|-------------|-----------------|-----------------------------------|
| 0 | 40 | 3045 | 13.1 |
| 1-4 | 10 | 11230 | 0.9 |
| 0-4 | 50 | 14275 | 3.5 |
| 5-9 | 3 | 15130 | 0.2 |
| 10-14 | 7 | 15780 | 0.4 |
| 15 - 19 | 17 | 15870 | 1.1 |
| 20 - 24 | 28 | 15800 | 1.8 |
| 25 - 29 | 25 | 13490 | 1.8 |
| 30 - 34 | 27 | 12650 | 2.1 |
| 35 - 39 | 39 | 13080 | 3.0 |
| 40 - 44 | 50 | 13920 | 3.6 |
| 45 - 49 | 103 | 12700 | 8.1 |
| 50 - 54 | 102 | 11900 | 8.6 |
| 55 - 59 | 143 | 9110 | 15.7 |
| 60 - 64 | 172 | 6240 | 27.6 |
| 65 - 69 | 233 | 4290 | 54.3 |
| 70 - 74 | 206 | 3000 | 68.7 |
| 75 - 79 | 214 | 1890 | 113.1 |
| 80 + | 506 | 1570 | 322.5 |
| Total | 1925 | 180695 | |

Summarizing the Mortality Level in a Population

- ◆ Each age-specific death rate is largely independent of age structure.
- ◆ Ideally, we would also like to have summary measures of mortality across broad ages that are independent of age structure.
- ◆ That is what the life table does for us. A life table presents the probability of a person dying at each age
- ◆ For instance, a key summary measure it provides is life expectancy at birth.
- ◆ We compute these indicators AFTER we have adjusted our death registration data for incompleteness
- ◆ Advantages of such measures:
 - ◆ Independent of age structure (unlike CDR)
 - ◆ Intuitive interpretation relates to the individual lifespan

INTRODUCTION

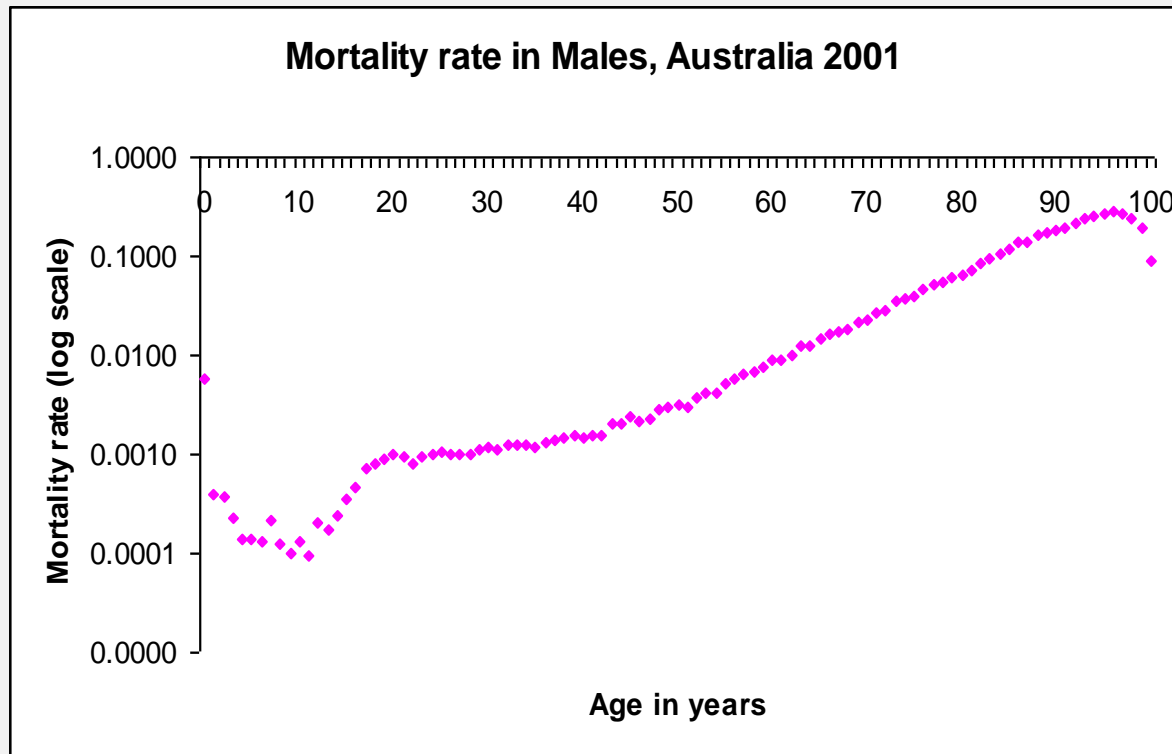
- ◆ Period analysis - analysis of a population:
 - ◆ for an *interval of time* (eg, the year 2000 or the period 2000-2004), or
 - ◆ at a *point in time* (eg, a Census in 2000)
- ◆ Cohort analysis
 - ◆ Birth cohort (eg, people born in 1990)
 - ◆ Usually calculated separately by gender

The period life table

- ◆ Constructed from mortality rates from different age groups observed during a given time period (e.g. a year)
- ◆ Provides a cross sectional view of mortality and survival experience
- ◆ Illustrates “...what *would happen* to a cohort *if* it were subjected for all of its life to the mortality conditions of that period” (Preston, S., Heuveline, P. & Guillot, M., (2001))
- ◆ It is what we call a *synthetic* or *hypothetical* cohort
- ◆ A more practical and useful tool for mortality analysis

The complete life table

- Is built up from data on deaths and population at each single year of age



The abridged life table

- ◆ Reason for abridged life tables is that there are usually insufficient numbers to do an analysis by year by gender.
- ◆ Based on assumption that death rates are similar at neighbouring ages
- ◆ Uses deaths rates calculated from groups of ages
- ◆ Most common groupings are <1, 1-4, 5-9, ... 95-99, 100+ (others are also used)
- ◆ Where populations are small – ten year groupings used

Do we have enough data?

- ◆ Are there results in our first and last age groups?
- ◆ Is there more than 5000 person years of data included?
- ◆ Do our graphs of age-specific mortality have a clear shape?



Calculating abridged life tables

| Age interval | Years in interval | Linearity Adjust-ment | Reported pop/ births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
|--------------|-------------------|-----------------------|----------------------|-----------------|----------------|----------------------|--------------------------|--------------------------|-------------------|-------------------------|----------------------------------|---------------------|
| (years) | n_x | a_x | N_x | $d_{(adj)}$ | m_x | q_x | p_x | l_x | d_x | L_x | T_x | e_x |
| <1 | 1 | 0.1 | 141158 | 678.1 | 0.00480 | 0.00478 | 0.99522 | 100000 | 478 | 99570 | 7828154 | 78.28 |
| 1-4 | 4 | 0.5 | 531025 | 153.0 | 0.00029 | 0.00115 | 0.99885 | 99522 | 115 | 397858 | 7728585 | 77.66 |
| 5-9 | 5 | 0.5 | 687357 | 88.0 | 0.00013 | 0.00064 | 0.99936 | 99407 | 64 | 496876 | 7330727 | 73.74 |
| 10-14 | 5 | 0.5 | 719258 | 86.0 | 0.00012 | 0.00060 | 0.99940 | 99343 | 59 | 496569 | 6833851 | 68.79 |
| 15-19 | 5 | 0.5 | 726266 | 446.0 | 0.00061 | 0.00307 | 0.99693 | 99284 | 304 | 495659 | 6337282 | 63.83 |
| 20-24 | 5 | 0.5 | 747927 | 621.1 | 0.00083 | 0.00414 | 0.99586 | 98980 | 410 | 493873 | 5841623 | 59.02 |
| 25-29 | 5 | 0.5 | 708376 | 695.1 | 0.00098 | 0.00489 | 0.99511 | 98570 | 482 | 491642 | 5347750 | 54.25 |
| 30-34 | 5 | 0.5 | 743386 | 799.1 | 0.00107 | 0.00536 | 0.99464 | 98087 | 526 | 489121 | 4856108 | 49.51 |
| 35-39 | 5 | 0.5 | 759543 | 957.1 | 0.00126 | 0.00628 | 0.99372 | 97561 | 613 | 486275 | 4366987 | 44.76 |
| 40-44 | 5 | 0.5 | 762579 | 1344.1 | 0.00176 | 0.00877 | 0.99123 | 96949 | 851 | 482616 | 3880712 | 40.03 |
| 45-49 | 5 | 0.5 | 741136 | 1788.2 | 0.00241 | 0.01199 | 0.98801 | 96098 | 1152 | 477609 | 3398096 | 35.36 |
| 50-54 | 5 | 0.5 | 679033 | 2250.2 | 0.00331 | 0.01643 | 0.98357 | 94946 | 1560 | 470827 | 2920487 | 30.76 |
| 55-59 | 5 | 0.5 | 636723 | 3393.3 | 0.00533 | 0.02630 | 0.97370 | 93385 | 2456 | 460787 | 2449660 | 26.23 |
| 60-64 | 5 | 0.5 | 496072 | 4223.4 | 0.00851 | 0.04168 | 0.95832 | 90930 | 3790 | 445173 | 1988872 | 21.87 |
| 65-69 | 5 | 0.5 | 385226 | 5691.6 | 0.01477 | 0.07124 | 0.92876 | 87140 | 6208 | 420178 | 1543699 | 17.72 |
| 70-74 | 5 | 0.5 | 302778 | 8290.8 | 0.02738 | 0.12814 | 0.87186 | 80932 | 10371 | 378731 | 1123522 | 13.88 |
| 75-79 | 5 | 0.5 | 252158 | 11004.1 | 0.04364 | 0.19674 | 0.80326 | 70561 | 13882 | 318100 | 744791 | 10.56 |
| 80-84 | 5 | 0.5 | 166000 | 11358.2 | 0.06842 | 0.29214 | 0.70786 | 56679 | 16558 | 242000 | 426691 | 7.53 |
| ≥85 | 14 | 0.5 | 104337 | 14453.5 | 0.13853 | 1.00000 | 0.00000 | 40121 | 40121 | 184691 | 184691 | 4.60 |

$x+nM_x$

Definition: age-specific death rates

$$M_x = \frac{\text{Deaths during the year of persons between age } x \text{ to } x + n}{\text{Population aged } x \text{ to } x + n}$$


Difference between

M_x - population based rate

and

q_x : - cohort rate (probability)

| Age interval | Years in interval | Linearity Adjust-ment | Reported pop/ births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
|--------------|-------------------|-----------------------|----------------------|-----------------|----------------|----------------------|--------------------------|--------------------------|-------------------|-------------------------|----------------------------------|---------------------|
| (years) | n_x | a_x | N_x | $d_{(adj)}$ | m_x | q_x | p_x | l_x | d_x | L_x | T_x | e_x |
| <1 | 1 | 0.1 | 141158 | 678.1 | 0.00480 | 0.00478 | 0.99522 | 100000 | 478 | 99570 | 7828154 | 78.28 |
| 1-4 | 4 | 0.5 | 531025 | 153.0 | 0.00029 | 0.00115 | 0.99885 | 99522 | 115 | 397858 | 7728585 | 77.66 |


$${}_nq_x$$

Definition: the probability of dying between ages x and $x + n$

$${}_nq_x = \frac{n * {}_nM_x}{1 + n(1 - {}_na_x){}_nM_x}$$

where ${}_na_x$ is the proportion of the interval lived by those who died

| Age interval | Years in interval | Linearity Adjust-ment | Reported pop/ births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expecte d) | Deaths (expect ed) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expect ancy |
|--------------|-------------------|-----------------------|----------------------|-----------------|----------------|----------------------|--------------------------|---------------------------|--------------------|-------------------------|----------------------------------|----------------------|
| (years) | n_x | a_x | N_x | $d_{(adj)}$ | m_x | q_x | p_x | l_x | d_x | L_x | T_x | e_x |
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Approximating q_x

- Accuracy depends on assumption that those who die in the year live, on average, half a year during that year
- Fairly accurate for all ages except the very young (tend to be concentrate in early part of first year) and the very old
- This fraction usually denoted by a_x
- It is convention to use ${}_1a_0=0.1$ in low mortality countries and 0.3 in high mortality countries
- For all ${}_4a_1$, use 0.4

| Age interval | Years in interval | Linearity Adjustment | Reported pop/ births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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q_{x+}

- ◆ The last age group (q_{x+}) is open ended
- ◆ All those alive at the beginning of the last age interval must die eventually
- ◆ q_{x+} always equals 1



| Age interval | Years in interval | Linearity Adjustment | Reported pop/births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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| (years) | n_x | a_x | N_x | $d_{(adj)}$ | m_x | q_x | p_x | l_x | d_x | L_x | T_x | e_x |
| 80-84 | 5 | 0.5 | 166000 | 11358.2 | 0.06842 | 0.29214 | 0.70786 | 56679 | 16558 | 242000 | 426691 | 7.53 |

 $n p_x$

Definition: probability of surviving between exact age x and $x+n$

$${}_n p_x = 1 - {}_n q_x$$

$$\therefore {}_n p_x + {}_n q_x = 1$$

| Age interval | Years in interval | Linearity Adjustment | Reported pop/births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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l_x

Definition: the number of people alive at exact age x .

- Refers to an exact age rather than age interval (unlike other columns in an abridged life table)
- The first value, l_0 , is an arbitrary number called the *radix* (usually a round number such as 1, or 1000, or 100,000)

| Age interval | Years in interval | Linearity Adjustment | Reported pop/births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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Calculating l_x

First choose a *radix* then work down the table using:

$$l_x = l_{x-n} \bullet_n P_{x-n}$$

- For the last (open-ended) interval, the number of persons dying is the same as the number alive at its start

| Age interval | Years in interval | Linearity Adjustment | Reported pop/births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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Interpreting l_x

- ◆ l_x is an ***hypothetical*** number of survivors from a cohort of ***hypothetical births***, assuming these imaginary births were to experience the same age-specific mortality rates as those in the population of interest
- ◆ l_x bears no relation to the ***actual number*** of individuals aged x to $x+n$ in the real population
- ◆ The l_x value is only meaningful when related to the original radix

$${}_nL_x$$

Definition: the total number of person-years lived (contributed by those alive and those who died) between exact ages x and $x+n$

- Each person who remains alive throughout a specified time interval contributes n years of life
- Each person who dies in an interval contributes a fraction of the interval equivalent to ${}_n a_x \cdot n$ years

| Age interval | Years in interval | Linearity Adjust-ment | Reported pop/ births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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Calculating ${}_nL_x$

$${}_nL_x = n(l_{x+n} + {}_n a_x \cdot n d_x)$$

| Age interval | Years in interval | Linearity Adjustment | Reported pop/births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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$$T_x$$

Definition: the total number of person-years lived after age x

- Is obtained by cumulating the ${}_nL_x$ function from the bottom (highest age interval) up
- Refers to exact age x , not an age interval

| Age interval | Years in interval | Linearity Adjustment | Reported pop/births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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e_x

Definition: expected (average) number of years of life left for a person aged x

- For the l_x people alive at age x , the total number of years left for them to live is given by T_x
- On average, each of these l_x individuals has approximately T_x/l_x years to live

$$e_x = \frac{T_x}{l_x}$$

| Age interval | Years in interval | Linearity Adjustment | Reported pop/births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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e_0

- ◆ The average number of years available to a new born is generally referred to as the ***expectation of life at birth*** and is given by the ratio:

$$e_0 = \frac{T_0}{l_0}$$

| Age interval | Years in interval | Linearity Adjustment | Reported pop/births | Adjusted deaths | Mortality rate | Probability of dying | Probability of surviving | Pop surviving (expected) | Deaths (expected) | Years lived in interval | Cumulative yrs lived in interval | LE: Life Expectancy |
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Model Life Tables

- ◆ Age-specific mortality data are not always available
 - ◆ ie, we do not have accurate vital registration data
- ◆ Life table of a similar population, such as a neighboring country, can be useful
- ◆ Alternatively, we can construct a model life table
- ◆ Model life tables are sets of hypothetical life tables spanning the full range of life expectancies as well as different patterns of age-specific mortality (Rowland 2003: 312)

Model Life Tables

- ◆ Ideally, model life tables should:
 - ◆ Be simple and easy to use
 - ◆ Adequately capture the range of mortality patterns observed in populations
 - ◆ Be a good predictor of true underlying death rates

Types of model life tables

1. UN Model Life Tables (1955)
2. UN Model Life Tables for Developing Countries (1982)
3. Coale and Demeny regional (East, North, West South) model life tables (1966, 1983)
4. Brass Logit Life-table System (1971)
5. WHO Modified Brass Logit System (2003)

Lab 1: Construct a life table

- ◆ Use: LifeTableProbDyingUNSW_v3.xlsx
- ◆ With the test data, make a pivot tables for deaths by age group for both males and females.
- ◆ Input the deaths for males and the male pop
 - ◆ using ages <1 to 85+ for males
- ◆ repeat for females